Extraction of Aerodynamic Derivatives from Flight Data, Using an Analog Regression Technique

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The continuous nature of flight-test data acquired in a derivative extraction program suggests that the analog computer can be employed in the data reduction process. Such an approach is feasible if an appropriate regression technique is employed. Using a conventional dynamic model for the aircraft, such a technique is presented herein. Linearization of the model is done for convenience, not necessity. Aerodynamic derivatives acquired in this program agree quite well with expected accuracies, although in some instances they differ considerably from predicted values. These are the cases we seek. The convenience and accuracy of the analog regression technique suggests possible inclusion of this procedure in standard flight-test evaluation programs.

Nomenclature

a_i, a_{ij} = elements of the regression matrices for the side-force equation
b_i, b_{ij} = elements of the regression matrices for the rolling moment equation
b = wing span of aircraft
c_i, c_{ij} = elements of the regression matrices for the yawing moment equation
C_l = rolling moment coefficient, L/qSb
C_n = yawing moment coefficient, N/qSb
C_y = side-force coefficient, mU_0Y/qS
G = regression criterion function to be minimized
g = gravitational constant, 32.2 ft/sec^2
I_x = mass moment of inertia about the longitudinal principal axis
I_z = mass moment of inertia about the vertical principal axis
L = rolling moment about the longitudinal axis
$\mathcal{L}$ = summation of terms in the rolling moment equation
m = mass of the aircraft
N = yawing moment about the vertical axis
$\mathfrak{N}$ = summation of terms in the yawing moment equation
n_y = lateral load factor on the aircraft
P = roll rate about the longitudinal axis
q = dynamic pressure on the aircraft
R = yawing rate about the vertical axis
S = wing area of the aircraft
t = time, during data gathering
U = component of aircraft velocity along its longitudinal axis
v = perturbation velocity along the aircraft's lateral axis
W = component of aircraft velocity along its vertical axis
Y = acceleration due to aerodynamic force along aircraft's lateral axis, divided by forward velocity
$\mathcal{Y}$ = summation of terms in the side-force equation
β = sideslip angle of the aircraft
δ = control surface deflection
ϵ_i = error in balancing force and moment equations using flight data
θ = pitch perturbation angle of the aircraft's longitudinal axis
τ = time, during analog calculations

Subscripts

a = aileron
0 = initial, constant conditions of the aircraft
r = rudder
1,2,3 = longitudinal, lateral, and vertical axes of the aircraft, respectively

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Introduction

THE mathematical treatment of statistical data is usually confined to digital computers since the nature of the data and the flexibility and accuracy of the computer make this equipment an appropriate choice. In the statistical extraction of aerodynamic derivatives from flight-test data some advantages can also be realized by employing analog equipment. This is particularly true if the method of analysis is in the time domain. Deferring the problem of instrumentation accuracy which arises in such a program, we find that the rapidity and convenience of the analog computer may even permit the inclusion of a derivative extraction program during standard evaluation tests for new aircraft. In general, the analog computer can perform this task efficiently in many instances where time is limited and particularly when the variables of the problem are continuous rather than discrete. Furthermore, with proper precautions, the accuracy of analog results can be quite comparable to those obtained with more sophisticated digital programs. We can say this because accuracy in this case is controlled largely by the quality of the test data rather than by choice of computational equipment.

The situation involved herein is one of obtaining aerodynamic derivatives on an operational fighter aircraft which in turn are to be used as design data for modifications to this aircraft. The method chosen to accomplish this is the familiar one of classical regression. This consists of finding coefficients or constants which relate a set of variables such that a selected error criteria (usually a function of the error squared) is minimized. This is frequently called a least square regression fit. For continuous variables such as those describing the perturbed motion of an aircraft, the problem can be handled in a straightforward manner on an analog computer.

Approach

The application of regression techniques to analog computers is discussed by G. Bekey in Ref. 1. A particularly informative paper by A. Rubin[2] discussed the use of an analog regression procedure for the problem of obtaining the best estimate of a missile miss-distance from a continuously telemetered signal transmitted by a target scorer system.

In the application discussed herein the well-known linear equations of motion serve as a mathematical model describing the motion of an aircraft perturbed from level flight. The longitudinal and lateral-directional modes, as usual are considered independent. Although aerodynamic derivatives have been obtained using both these modes, only the lateral-directional case is considered here. It should be noted that the quality of results obtained using the lateral-directional equations is comparable to that obtained with the longitudinal model. The former, however, appears more interesting since less success is usually experienced in determining lateral-directional derivatives than say, in obtaining normal force and pitching moment derivatives. Furthermore, the problem of linear dependency does not occur in the manner that it does in the longitudinal case.

We are dealing with lateral-directional equations written for an aircraft principal axis system which have been linearized for convenience, not necessity. These equations are

$$\begin{aligned}\dot{\beta} - (W_0/U_0)P + R - (g/U_0)\cos\theta_0 \textstyle\int Pdt - \\ (g/U_0)\sin\theta_0 \textstyle\int Rdt - Y_\beta\beta - Y_{\delta_r}\delta_r - Y_{\delta_a}\delta_a &= 0 \\ -L_\beta\beta - L_R R - L_{\delta_a}\delta_a - L_{\delta_r}\delta_r - L_P P + I_x\dot{P} &= 0 \\ -N_\beta\beta - N_R R - N_{\delta_a}\delta_a - N_{\delta_r}\delta_r - N_P P + I_z\dot{R} &= 0\end{aligned} \tag{1}$$

Employing flight derived motion data for β, P, R, δ_r, etc., we expect Eqs. (1) to develop the errors

$$\mathcal{Y} = \epsilon_1 \qquad \mathcal{L} = \epsilon_2 \qquad \mathcal{N} = \epsilon_3 \tag{2}$$

The selected optimization criteria is the minimization of the quantity defined as

$$G = \int_0^t (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)dt \tag{3}$$

where t represents the time interval over which significant motion data are acquired. Choosing τ to represent an independent analog computational time scale we have

$$\frac{dG}{d\tau} = \int_0^t \left(2\epsilon_1\frac{d\epsilon_1}{d\tau} + 2\epsilon_2\frac{d\epsilon_2}{d\tau} + 2\epsilon_3\frac{d\epsilon_3}{d\tau}\right)dt \tag{4}$$

where

$$\begin{aligned} d\epsilon_1/d\tau &= \partial\epsilon_1/\partial Y_\beta\, dY_\beta/d\tau + \partial\epsilon_1/\partial Y_{\delta_a}\, dY_{\delta_a}/d\tau + \dots \\ d\epsilon_2/d\tau &= \partial\epsilon_2/\partial L_\beta\, dL_\beta/d\tau + \partial\epsilon_2/\partial L_P\, dL_P/d\tau + \dots \\ d\epsilon_3/d\tau &= \partial\epsilon_3/\partial N_\beta\, dN_\beta/d\tau + \partial\epsilon_3/\partial N_P\, dN_P/d\tau + \dots\end{aligned} \tag{5}$$

The partial derivatives are evaluated

$$\begin{aligned} &\partial\epsilon_1/\partial Y_\beta = -\beta \qquad \partial\epsilon_1/\partial Y_{\delta_a} = -\delta_a \qquad \text{etc.} \\ &\partial\epsilon_2/\partial L_\beta = -\beta \qquad \partial\epsilon_2/\partial L_P = -P \qquad \text{etc.} \\ &\dots \qquad \text{etc.} \qquad \dots\end{aligned} \tag{6}$$

Rewriting Eq. (4) we find

$$\begin{aligned}\frac{dG}{d\tau} = \int_0^t 2\epsilon_1\left(\frac{\partial\epsilon_1}{\partial Y_\beta}\frac{dY_\beta}{d\tau}\right)dt + \int_0^t 2\epsilon_1\left(\frac{\partial\epsilon_1}{\partial Y_{\delta_a}}\frac{dY_{\delta_a}}{d\tau}\right)\times \\ dt + \dots + \int_0^t 2\epsilon_2\left(\frac{\partial\epsilon_2}{\partial L_\beta}\frac{dL_\beta}{d\tau}\right)dt + \int_0^t 2\epsilon_2 \times \\ \left(\frac{\partial\epsilon_2}{\partial L_P}\frac{\partial L_P}{d\tau}\right)dt + \dots + \int_0^t 2\epsilon_3\left(\frac{\partial\epsilon_3}{\partial N_\beta}\frac{dN_\beta}{d\tau}\right)dt + \dots\end{aligned} \tag{7}$$

Forcing $dG/d\tau$ always to be nonpositive permits G to be driven to a local minimum in a manner suggested in Ref. 3. Thus, we let

$$\begin{aligned}\frac{dY_\beta}{d\tau} &= -\int_0^t \epsilon_1\left(\frac{\partial\epsilon_1}{\partial Y_\beta}\right)dt = \int_0^t \epsilon_1\beta dt \\ \frac{dY\delta_a}{d\tau} &= -\int_0^t \epsilon_1\frac{\partial\epsilon_1}{\partial Y_{\delta_a}}dt = \int_0^t \epsilon_1\delta_a dt \\ \frac{dL_\beta}{d\tau} &= -\int_0^t \epsilon_2\frac{\partial\epsilon_2}{\partial L_\beta}dt = \int_0^t \epsilon_2\beta dt \\ \frac{dN_\beta}{d\tau} &= -\int_0^t \epsilon_3\frac{\partial\epsilon_3}{\partial N_\beta}dt = \int_0^t \epsilon_3\beta dt \\ &\dots \qquad \text{etc.} \qquad \dots\end{aligned} \tag{8}$$

Expanding $dY_\beta/d\tau$ in (8) yields

$$\begin{aligned}\frac{dY_\beta}{d\tau} = \int_0^t \beta\left(\dot{\beta} - \frac{W_0}{U_0}P + R - \frac{g\cos\theta_0}{U_0}\int_0^t Pdt - \right. \\ \left. \frac{g\sin\theta_0}{U_0}\int_0^t Rdt\right)dt - Y_\beta\int_0^t \beta^2 dt - \\ Y_{\delta_r}\int_0^t \beta\delta_r dt - Y_{\delta_a}\int_0^t \beta\delta_a dt\end{aligned} \tag{9}$$

and similarly for the other derivatives in (8). The integrals are computed using appropriate portions of flight transients and the known constants W_0, U_0, θ_0, etc. For the complete lateral-directional system we then have three sets of first-order differential equations in the time variable τ:

$$\begin{bmatrix}\dot{Y}_\beta \\ \dot{Y}_{\delta_a} \\ \dot{Y}_{\delta_r}\end{bmatrix} = \begin{bmatrix}a_1 \\ a_2 \\ a_3\end{bmatrix} + \begin{bmatrix}a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33}\end{bmatrix}\begin{bmatrix}Y_\beta \\ Y_{\delta_a} \\ Y_{\delta_r}\end{bmatrix} \tag{10}$$

$$\begin{bmatrix}\dot{L}_\beta \\ \dot{L}_P \\ \dot{L}_R \\ \dot{L}_{\delta_a} \\ \dot{L}_{\delta_r}\end{bmatrix} = \begin{bmatrix}b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5\end{bmatrix} + \begin{bmatrix}b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & \dots & \dots \\ b_{31} & b_{32} & \dots & \dots & \dots \\ b_{41} & \dots & \dots & \dots & \dots \\ b_{51} & \dots & \dots & \dots & \dots\end{bmatrix}\begin{bmatrix}L_\beta \\ L_P \\ L_R \\ L_{\delta_a} \\ L_{\delta_r}\end{bmatrix} \tag{11}$$

$$\begin{bmatrix}\dot{N}_\beta \\ \dot{N}_P \\ \dot{N}_R \\ \dot{N}_{\delta_a} \\ \dot{N}_{\delta_r}\end{bmatrix} = \begin{bmatrix}c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5\end{bmatrix} + \begin{bmatrix}c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{21} & c_{22} & c_{23} & \dots & \dots \\ c_{31} & c_{32} & \dots & \dots & \dots \\ c_{41} & \dots & \dots & \dots & \dots \\ c_{51} & \dots & \dots & \dots & \dots\end{bmatrix}\begin{bmatrix}N_\beta \\ N_P \\ N_R \\ N_{\delta_a} \\ N_{\delta_r}\end{bmatrix} \tag{12}$$

The analog computer is programmed to perform two operations in sequence. First, the integration of flight data to obtain the matrices defined by the coefficients a_{ij}, b_{ij}, and c_{ij}. Secondly, each of the three sets of simultaneous equations are solved for the condition where $\tau \rightarrow \infty$.

The L_R and N_P derivatives were assumed known since it was found that a more stable and repeatable solution of the L and N equations could thus be obtained. A further reduction in the size of the matrices is also achieved by the usual procedure of operating only one control surface during each flight run.

It is desirable to operate the aircraft with any existing damper modes and mechanical or electrical control cross-feeds disengaged. Although such procedure is not necessary, it is usually preferred since this permits flight test derivatives to be more easily compared to derivatives obtained by previous estimates or from wind-tunnel tests.

Acquisition and Reduction of Flight Data

Airborne Data

The aircraft in question was operated between 2,000 and 35,000 ft of altitude within a Mach number range of 0.30 to 0.90. Tests were conducted only in relatively smooth air. The aircraft was perturbed by either pulsing or stroking the ailerons or rudder by amounts sufficient to keep the ensuing motion within the range of linear aerodynamics. Stroking was done at a frequency of 5 rad/sec which is sufficiently above the short period frequency of the aircraft to avoid excessive excursions and also significantly below the frequency of the lowest structural mode. The stroking data proved excellent for evaluating the control effectiveness derivatives.

The acquisition of motion parameters P, R, δ_r, $\dot{P}$, etc. was accomplished by recording the output of sensors such as rate gyros, accelerometers, and actuator follow-up potentiometers directly onto an airborne magnetic tape in binary format (pulse code modulated). The binary tape was decoded and played onto a 14 track analog playback tape by processing through a digital to analog data processor which

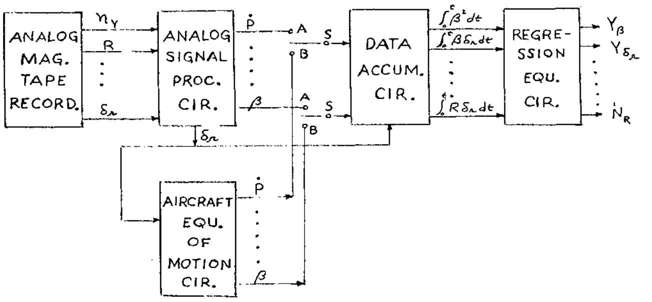

Fig. 1 Schematic of analog procedure for aerodynamic derivative extraction.

also applied static calibrations to each information channel. The airborne digitized data were recorded at 40 cps over a zero to 5-v range with a least significant bit of 20 mv. The 5-v range was reduced to 1.4 v, however during processing onto the analog tape in order to accommodate the range of the analog tape recorder. The processed analog tape contained the following information: 1) side-load factor near the center of gravity; 2) side-load factor 17 ft forward of the center of gravity; 3) normal-load factor near the center of gravity; 4) normal-load factor 17 ft forward of the center of gravity; 5) normal-load factor in the wing wheel well; 6) roll rate; 7) pitch rate; 8) yaw rate; 9) right aileron deflection; 10) left aileron deflection; 11) rudder deflection; 12) bank displacement; 13) pitch displacement; and 14) two seconds before start of run marker.

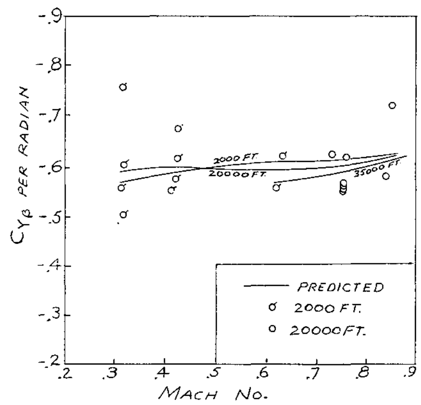

Fig. 2 Comparison of predicted and flight acquired values of the side-force derivative C_{Y_β}.

The normal- and side-load factors were measured by linear accelerometers and corrected to the instantaneous center of gravity of the aircraft by algebraic methods performed in the analog computer. Angular velocities were acquired from the outputs of rate gyros, which in turn were differentiated in the analog computer to obtain angular accelerations. Surface deflections were derived from potentiometers at the servoactuators. Pitch, bank, and yaw data were taken for reference purposes only since these functions were estimated by integration of the corresponding rates. A marker was played onto the analog tape at least 2 sec before actuation of the control surfaces to assist in calculation of the bias level in each information channel as discussed below.

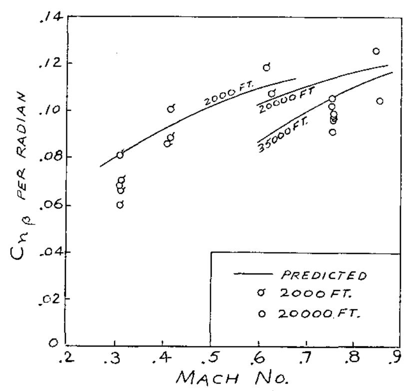

Fig. 3 Comparison of predicted and flight acquired values of the yawing moment derivative C_{n_β}.

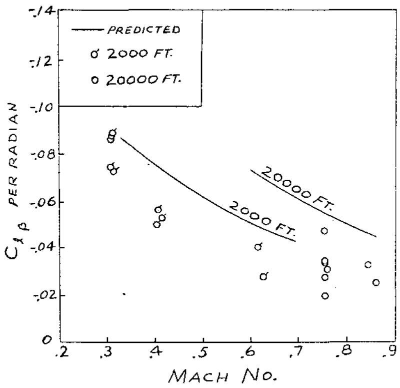

Fig. 4 Comparison of predicted and flight acquired values of the rolling moment derivative C_{l_β}.

Angle of Sideslip

The uncertainty of acquiring suitable static and dynamic calibrations for existing angle-of-attack or angle of sideslip sensors suggested that these functions might more reliably be calculated from existing rate and accelerative measurements. Pursuing this we have

$$\dot{v} = gn_r - RU_0 + PW_0 + g\sin\theta_0 \int_0^t R dt + g\cos\epsilon_0 \int_0^t P dt \quad (13)$$

$$\dot{\beta} \cong \dot{v}/U_0 \quad (14)$$

$$\beta = \int_0^t \dot{\beta} dt \quad (15)$$

The limits of integration are selected to cover the range of data of interest. In almost all cases the lower limit is taken as the aircraft's trim condition existing just before actuation of the control surfaces. It should be noted that a procedure similar to the aforementioned is followed for obtaining the angle of attack and the linear acceleration of the longitudinal axis in the analysis of longitudinal data.

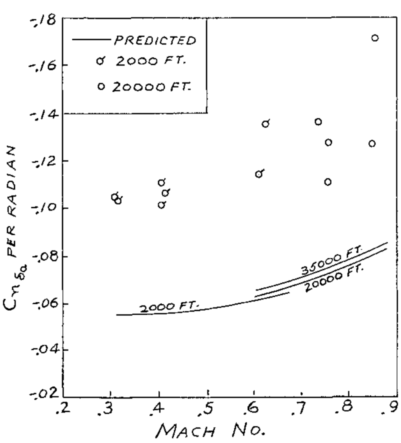

Fig. 5 Comparison of predicted and flight acquired values of the yawing moment derivative $C_{n_{\delta_a}}$.

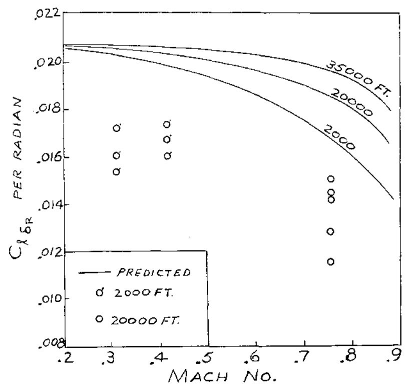

Fig. 6 Comparison of predicted and flight acquired values of the rolling moment derivative $C_{l_{\delta_R}}$.

Filtering Biasing and Phase Correction

The differentiation of angular rates has the advantage of assuring acceleration data with a minimum phase error between the two functions. Because of the digitized nature of the rate data, excessively noisy information results from differentiation of this signal. Filtering is quite effective, but in order to avoid the accompanying phase shift it is necessary to filter all information channels through a similar band pass.

It was noted that serious errors in results can occur by using information from channels having small relative phase displacements; consequently the selection of sensors with comparable frequency responses particularly near the aircraft's short period frequency is quite important. In an independent error study it was found that a phasing error of 5° between a rate and displacement signal (i.e., 85° instead of 90°) at the aircraft's short period natural frequency may easily produce errors of 20% in certain derivatives. Since the phasing problem was not thoroughly studied, certain precautions were taken once the test stand frequency responses of the airborne sensors were known. As a result it was felt wise to phase lag the output of the normal accelerometers in order to bring this information into closer timing with the other accelerometers and with the rate gyros.

The double integration needed to obtain the sideslip angle β in Eqs. (13–15) requires the loading of analog accumulators over a period of 5 to 10 sec. This process immediately reflects the influence of small bias in the input data by the failure of the sideslip angle to return to zero after all perturbations have subsided. The magnitude of this drift can be appreciably reduced by adjusting v as close to zero as possible at the initial conditions for each run. To do this a 2-sec period is selected immediately prior to control actuation in which acceleration and rate sensors and surface positions are sampled for mean values. That is

$$\bar{R} = \frac{1}{2}\int_0^2 R dt \qquad \bar{P} = \frac{1}{2}\int_0^2 P dt \qquad \text{etc.} \tag{16}$$

These mean values are applied as biases to the corresponding quantities in the calculation of $\dot{v}$ and are also applied as corrections to all places in the regression matrices where these functions occur.

Data Processing

Figure 1 schematically shows the procedure for extracting the aerodynamic derivatives from analog magnetic tape information.

With the switch S in the A position, the analog signal processing circuits receive sensor data from the tape recorder and perform the following operations:

1) Amplification of the tape signals by the input buffer amplifiers to accommodate the operating levels of the analog computer.

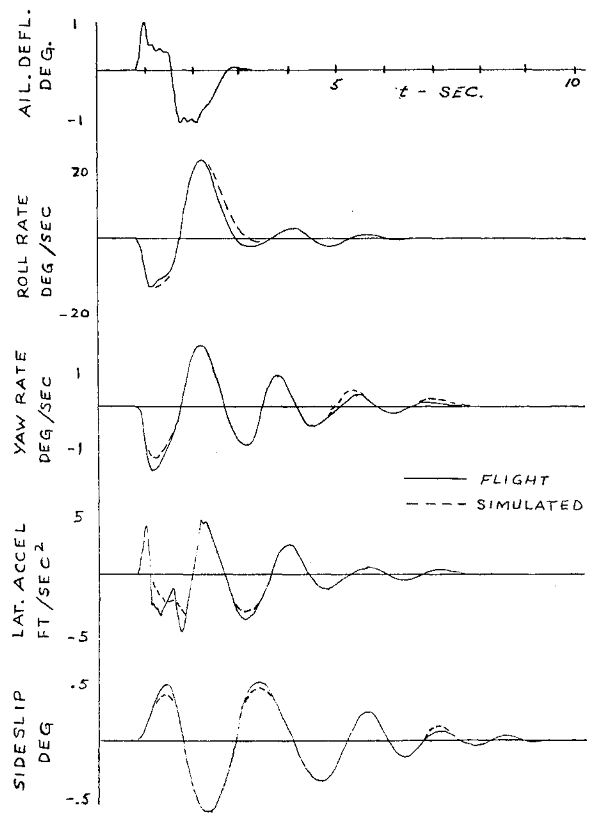

Fig. 7 Comparison of flight-test and analog simulated aircraft responses to identical aileron deflections.

2) Bias removal as indicated by Eq. (16).

3) Transformation of linear accelerometer data to obtain the acceleration at the instant center of gravity.

4) Computation of the angle of sideslip as indicated by Eqs. (13–15).

5) Differentiation of angular rates to obtain angular accelerations. Also filtering all data channels to reduce noise levels.

The data accumulation circuits generate the elements of the regression matrix. These elements consist of products and integrals of the sensed motion parameters in the form indicated by Eq. (9). Once the matrix elements are available, the regression equation's circuits immediately solve the first-order differential equations for the dimensional aerodynamic derivatives. These in turn can be reduced to nondimensional form from knowledge of the flight conditions under which the data were acquired.

Provisions are made for comparison checks between the actual flight transients and those obtained from the derived aerodynamic derivatives. These derivatives are introduced into the aircraft's equation of motion circuits which in turn are forced by the airplane's control surface motion. Placing switch S in the B position permits intermittent checks on the regression process.

Expected Accuracies

A mathematically acceptable error analysis yielding the accuracies to be expected from a program such as described previously would be quite difficult and probably impractical since the effort required would be comparable to that needed in solving the original problem. Some acceptable means of combining the effects of all possible errors would be necessary. This is further aggravated by the fact that the effect of individual errors is asymmetrical in that plus and minus errors produce different deviations in the results.

To circumvent these difficulties the subject aircraft was simulated in the analog computer and responses determined for known control inputs. The regression technique was applied to the responses and aerodynamic derivatives extracted for comparison with the original input data. Subsequently, the responses were modified to represent expected instrument errors, biases, phasing errors, noise, etc. Errors introduced by digital to analog data processing were known from previous experience with this equipment. Accumulating the results indicated that for each derivative, 68% of the extracted data are expected to deviate 10% or less from its mean value.

Results

Figures 2–6 are typical of the results obtainable with the analog regression technique. In general, the scatter of the reduced data is consistent with the expected accuracy of these results. The predicted curves in these figures were obtained by correcting wind-tunnel data for airframe aeroelastic effects. The sideslip derivatives $C_{Y\beta}$, $C_{n\beta}$, and $C_{l\beta}$ in Figs. 2–4 show very good agreement with these predictions. Of greater interest to the designer however, are those cases in which predicted results are not upheld. Figures 5 and 6 show such cases for the derivatives $C_{n\delta_a}$ and $C_{l\delta_R}$. Both sets of results indicate well grouped data following the expected trends although their magnitudes are considerably different from predicted values. It is results of this type which we seek since they redefine the previously accepted aerodynamic characteristics of the aircraft. It is immediately clear that as a consequence of such results re-evaluation may be required in such areas as automatic control, airloads, and flying qualities. In Fig. 7, the aircraft's response to an aileron pulse is compared directly to a simulated response using the same control input and extracted aerodynamic derivatives. Such comparisons indicate the fidelity of the mathematical model as constructed with flight derived aerodynamic derivatives.

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